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MACCOLL ON JUDGEMENT AND INFERENCE*

The first part of the paper presents a framework of distinctions for the philosophy of logic in which the interrelations between some central logical notions, such as judgement(-act), judgement (made), proposition(al content), consequence, and inference are spelled out. In the second half the system of MacColl is measured against the distinctions offered in the framework.

The theme of our conference is that of Hugh MacColl and the logical tradition. From any point of view, surely, judgement and inference are (possibly *the*) central components of the logical tradition. However, they do not occur as such in MacColl's *Symbolical reasoning*(s). What we find are statements, assertions and applications of the symbol $\therefore$. Accordingly, I begin with a rational reconstruction of what I see as the pivotal moment in the 19th century logical tradition, namely Bolzano's introduction of a novel form of judgement, which will be used to take the measure of the early MacColl with respect to judgement and inference.

I. A Logical Framework

Hilary Putnam and, following him, George Boolos have, on different occasions, taken exception to Quine's dictum that

"*Logic is an old subject, and since 1879 it has been a great one*",

with which he opened the first editions of his *Methods of Logic*.[1] In their opinion, Quine's implicit preference for Frege's *Begriffsschrift* does an

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[1] Putnam 1982, Boolos 1994, and Quine 1950.

injustice to Boole (Boolos and Putnam) and the Booleans, of whom Peirce in particular (Putnam). Ten years ago, in an inaugural lecture at Leyden, I too argued that Quine presented too narrow a view of logic, and that as far as the nineteenth century was concerned the crucial date in the development of logical doctrine was not 1879 (nor 1847, I would add today, disagreeing with Boolos's stimulating paper), but 1837, the year in which Bernard Bolzano published his treatment of logic in four hefty volumes.[2]

Why does this Bohemian priest deserve pride of place over and above such luminaries as Boole, Peirce and Frege? For more than two thousand years, logic has been concerned with how to effect valid acts of inference from judgements known to other judgements that become known through the inference in question. Basically, these judgements take the subject/copula/predicate form [S is P]. Bolzano now has the courage to break with this traditional pattern and uses instead the unary form

(1) A is true,

where A is a *Satz an sich*, or a *Gedanke*, in the later alternative terminology of Frege. The latter term was translated into English as *proposition* by Moore and Russell, with an unusually confusing ambiguity as a result: prior to 1900 a "proposition" stood for a judgement (made), whereas later it came to stand for the propositional content of such a judgement. For Bolzano, logic was very much concerned with knowledge; his critical examination and exposition of *logic* is called *Wissenschaftslehre* [an approximate translation might be *The Theory of (Scientific) Knowledge*]. Just as his main target, Kant, he holds that a correct (*richtig*) judgement is a piece of knowledge (*eine Erkenntnis*).[3] To my mind, he is perfectly right in doing so. After the "linguistic turn", in place of judgements, one can consider instead the proper form, and relevant properties, of their linguistic counterparts, namely assertions. An assertion is effected by means of the assertoric utterance of a declarative sentence. This explanation must be supplemented with a criterion of assertoric force, on pain of a vicious circularity. Such a criterion is provided by means of the question:

(2) How do you know? What are your grounds?

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[2] *Oordeel en Gevolgtrekking. Bedreigde Species?*, an inaugural lecture delivered at Leyden University, September 9, 1988, and published in pamphlet form by that university.

[3] Bolzano 1837, § 34.

which is legitimate as a response to an assertion. In case the utterance was assertoric, the speaker is obliged to answer, and if he cannot do so the assertion was *blind*. The assertion issued by an act of assertion, but for what is stated, also contains an illocutionary claim to knowledge. Thus, I am able to make public my knowledge that snow is white through the assertoric utterance of the declarative

(3) 'Snow is white'.[4]

The explicit form of the assertion thus made would then be:

(4) I know that snow is white.

A sole utterance of the nominalization

(5) that snow is white,

which expresses the propositional content, on the other hand, will not so suffice. A phrase sufficient for the making of assertions is reached by appending 'is true' to the nominalization in question. An assertoric utterance of the declarative

(6) that snow is white is true

does suffice to effect an assertion with (4) as the assertion made. The fully explicit form, with indication of knowledge, and truth of content, accordingly becomes:

(7) I know that that snow is white is true.

This grammatically necessary, but hardly idiomatic, iteration of *that* can be avoided here through the transformation:

(8) that S is true = it is true that S,

which yields

(9) I know that it is true that snow is white

as the explicit form of the assertion made through an utterance of (3).

I do not mean to imply that this was the route that Bolzano actually took to his novel form of judgement: it was not. I have used various linguistic considerations concerning the form of assertions when viewed as reports of knowledge, whereas Bolzano insisted that his *Sätze an sich*

[4] The example 'snow is white' is taken from Boole (1854, p. 52).

were completely independent of all matters linguistic and cognitive. Be that as it may, the argument given provides a rationale for why correct judgements made are pieces of knowledge, and why the proper form of judgement is "truth ascribed to propositional content".

There remains the problem of choosing appropriate terminology for entities in the expanded declarative form (6). Frege held that declarative sentences expressed *propositions*, that is, for example, the declarative *snow is white* expresses the proposition that snow is white. I prefer not to join Frege in this. Wittgenstein used the terminology *Satz* and *Satzradikal*. The latter, clearly, is the proposition(al content), but the former has to do double duty for declaratives and what they express. For my purposes the best choice here might well be *statement*.[5] Sentence, statement and proposition then serve in different logical roles:

(10) a declarative sentence expresses
a statement with a proposition as content.

Thus,

(11) the declarative 'snow is white' expresses that snow is white.

When one steps over to the expanded statement-form an iteration of *that* occurs,

(12) 'snow is white' expresses that that snow is white is true,

which can be removed using the transformation (8)

(13) the declarative 'snow is white' expresses
that it is true that snow is white.

[5] *Enunciation* and *declaration* are other alternatives. The latter has a certain high-sounding ring to it, but might otherwise have served very well. My discussion could then have been pithily summarised:

The assertoric utterance of a declarative makes an assertion that claims knowledge of the declaration expressed (declared?) by the declarative in question.

My preference for statement is, i.a., based on the fact that *statement* is the English term which is applied to reports by witnesses. This is followed also in German: *Aussage, Zeugnis*, and Swedish: *utsaga*, whereas Dutch: *Declaratie, verklaring* uses *declaration* instead. Also MacColl (1880, p. 53) links his use to the legal one, for which reference I am indebted to Shahid Rahman. This advantage has to be weighed against the drawback that since Cook Wilson—*Statement and Inference*—the term has been in constant Oxford use, where it has served in many roles, among which those of propositional content (with indexicality taken into account; perhaps, after E. J. Lemmon (1966), the most common current use), declarative sentence, and state of affairs, the act of saying, and what is said, the act of asserting, and the assertion made.

Consider now an act of assertion made through an assertoric utterance of the declarative sentence 'snow is white'. With respect to the assertion made the discussion above can be summarised in the following table:

Assertion made (explicit form)	I know that it is true that snow is white.
(Illocutionary) Knowledge-claim	I know that
Statement that is asserted	that snow is white is true = it is true that snow is white
Propositional content	that snow is white

Note here also that judgement is often used instead of *statement* and *assertion*, with respect to both the act and the object. Thus propositions have truth-conditions, whereas statements (judgements) have assertion-conditions.

The implication A implies B, in symbols $A \supset B$, between two propositions A and B, is another proposition, which accordingly is a candidate for truth. Classically $A \supset B$ is true when A is false or B is true, whereas its constructive truth consists in the existence of a suitable proof-object. It should be stressed that 'implies' can only join propositions, but not statements: the proposition that grass is green implies that snow is white is fine from a grammatical point of view, whereas an attempted connection between statements yields the nonsensical 'grass is green implies snow is white', which, as Quine noted (1940), contains too many verbs. Propositions can also be joined into a relation of consequence, which yields a generalisation of propositions:

(14) the consequence from A to B,

in (Gentzen-like) symbols $A \Rightarrow B$.[6]

The consequence, or sequent, $A \Rightarrow B$ holds precisely when the corresponding implication $A \supset B$ is true (also constructively). Much to his credit, Bolzano considered also this notion of consequence—he called it *Ableitbarkeit*—whereas today one is only interested in the *logical* holding of the consequence. (A consequence holds logically when the corresponding implication is a logical truth, that is, is true come what may, independently of what is the case.) It should be clear that the inference

$$\frac{A \Rightarrow B \text{ holds} \quad A \text{ is true}}{B \text{ is true}}$$

[6]Consequences between statements will not work for the Quinean reasons. Cf. the preceding footnote.

is perfectly valid as it stands; one does not need the logical holding or the logical truth in the premises in order to be allowed to conclude that B is true. (Similarly, we do not need the *logical* truth of $A \supset B$ in order to draw the conclusion that B is true from the premise that A is true.)

Just as we can combine propositions into both implications, which are propositions, and consequences, which are not, statements can be combined into conditionals, which are statements, and inferences, which are not. For example, a *conditional statement* results from applying, not categorical, but hypothetical truth

(15) $\ldots$ is true, provided that A is true,

to a proposition:

(16) B is true, provided that A is true.

The *proviso* can also be expressed in other ways: on condition that, under the hypothesis that, assuming that, etc., will all serve equally well here. Conditional statements can be obtained also in other ways; for example, by joining statements by means of *If-then*:

(17) If A is true, then B is true.

The assertion-conditions for the three statements

$A \supset B$ is true,

$A \Rightarrow B$ holds,

B is true, provided that A is true, or, in another formulation,

If A is true, then B is true

are different (we do not have the same statement three times over), but if one is entitled to assert any one of them, the requirements for asserting the others can also be met.

Finally, an inference is, in the first instance, a mediate act of judgement, that is, (taking the linguistic turn) an act of asserting a statement on the basis of other statements being already asserted (known). So the general form of an inference I is:

$$\frac{J_1 \ldots J_k}{J.}$$

The inference I is valid if one is entitled to assert J when one knows (has asserted) $J_1 \ldots J_k$. Accordingly, in order to have the right to draw the inference I must possess a chain of immediately evident axioms and inferences that link premises to conclusion.[7] After Bolzano it has been common to conflate the validity of the inference I′

$$\frac{A_1 \text{ is true}, \ldots, A_k \text{ is true}}{C \text{ is true.}}$$

with the logical holding of the consequence $A_1, \ldots, A_k \Rightarrow C$. That is, one reduces the validity of the inference to the logical holding of a relation of consequence between the propositional contents of statements that serve as premises and conclusion, respectively, of the inference in question. Bolzano also reduced the correctness of the statement that the rose is red is true to the rose's *really* being red. In both cases, the reduction gives rise to what Brentano called *blind judgements*: a judgement can be correct, by fluke, even though the judger has no grounds, and similarly for blind inference.

Bolzano's other notion of consequence—that of *Abfolge*—is less clear, but can perhaps be understood in the following way. Consider the inference

(18) S_1. Therefore: S_2.

In expanded form it becomes:

(19) That S_1 is true. Therefore: that S_2 is true.

When this inference is drawn and made public through an utterance of (18), we have assertions of (i) the premise that S_1 is true, (ii) of the conclusion that S_2 is true, and (iii) of the inferential link between them. Instead of considering the validity of the inference, Bolzano's *Abfolge* involves a *propositional operator* ... entails ... such that

(20) The proposition that S_1 entails that S_2 is true =
the inference (18) is valid, and
the premise *that S_1 is true* is correct.

II. The Early Measure of MacColl

How does Hugh MacColl stand with respect to the novel Bolzano-form of judgement? How do his writings fare when measured against the above battery of distinctions? The early parts of his *Symbolic(al)*

[7]This notion of validity is age-old. Compare Quine and Ullian (1970, p. 22) for a recent formulation: 'When a ... truth is too complicated to be appreciated out of hand, it can be proved from self-evident truths by a series of steps each of which is itself self-evident—in a word it can be deduced from them.'

Reasoning(s) are in many ways as fascinating and as difficult as the corresponding passages in Frege's *Begriffsschrift*.[8] The logical key-term on which they rest is that of a *statement*: 'My system, which adopts full and complete *statements* as the ultimate constituents into which any argument can be resolved, steers clear of the discussion altogether.'[9] Some would expect a careful definition, or (perhaps better) elucidation, of the notion in question, according to which other claims could then be made evident owing to the meaning assigned to the term *statement*. Such, for instance, is, *mutatis mutandis*, Frege's procedure in the *Grundgesetze*. Frege, indeed, is one of the foremost proponents of the paradigm that Jean van van Heijenoort (1967) has dubbed "Logic as Language" and Hintikka (1988, 1996) has transformed into "Language as the Universal Medium". It is hard to determine MacColl's position with respect to this paradigm: like the Booleans he seems to appreciate the calculus aspects more than the language aspects, but, on the other hand, his remarks concerning pure and applied logic (1880, pp. 48, 58) point in the direction of the Logic as Language conception. Also the remarks concerning the *Law of Implication* (1880, p. 52), with their strong epistemological slant, tend in the direction of Logic as Language, as do the considerations on logical methodology (1880, p. 59). For sure, MacColl does not offer an explicit definition of the notion of statement. Instead he confines himself to examples, and other indications, of the roles in which his statements serve. (In this, of course, he is no worse than, say, Bolzano with respect to his *Sätze an sich*.) Statements are the denotations of the *'temporary symbols'*, that is, variables, or, perhaps even better, 'statement letters' in modern terminology. Temporary symbols can be joined into more complex ones by means of certain 'permanent symbols', which denote relations in which statements stand, and accordingly play the roles of logical constants in current logical formalisms. Examples (1880, p. 49) of statements are: "He is tall", "He is dark", and "He is German"; when a, b and c denote these statements respectively, their logical product abc denotes the statement "He is a tall, dark German." Furthermore, statements are *made*, or so MacColl informs us in definitions 1 and 2 (1880, pp. 49–50).

[8]In preparing my talk I have had to confine myself largely to MacColl's series of articles in *Mind*, owing to the difficulty in obtaining copies of further works. It should be noted that its book-length compilation from 1906 differs substantially from the earlier articles of the series.

[9]MacColl 1880, p. 59. The discussion in question concerns Hamilton's quantification of the predicate. The point concerning the use of statements is reiterated in MacColl 1902, p. 352, where propositions are also explained as subject-predicate statements.

Complex statements are obtained from other statements by means of (iterated) applications of

$$\times \text{ and } +,$$

which give, respectively, *compound statements* and *disjunctive statements*. Which notion, if any, of *my* proposition, statement and assertion corresponds best with MacColl's notion of a statement? MacColl's use of the "permanent symbols" $\times$, $+$, $'$, as well as many of the resulting formulae (1880, p. 53), is strongly reminiscent of modern uses of the corresponding propositional operators that are well known since Boole and Frege. Thus, the statements of MacColl would correspond to the *Sätze an sich/Gedanken* of Bolzano and Frege, and through *Principia Mathematica*, to the wff's of all of modern mathematical logic.

On the other hand, MacColl tells us,

> [t]he disjunctive symbol $a + b + c$ asserts that one of the three events named will take place, but it makes no assertion as to whether or not more than one will take place.

Here a disjunctive symbol, which "denotes" (expresses?) a disjunctive statement, appears to function as an *assertion*. Similarly, a denial a' is held to *assert* in definition 5 (1880, p. 52), and in the formulae (1.) $aa' = 0$ and (2.) $a + a = 1$, the accent seems to function as a propositional negation-sign. In formula (3),

$$(abc\dots)' = a' + b' + c' + \dots ,$$

on the other hand, if we read the $+$ and $'$ with assertoric force, according to their explanations, we get manifold violations of Geach's (1965) *Frege-point*: $(abc)'$ is assertoric and stands in the antecedent of an implication: $(a = b) =_{\text{def.}} (a : b) \times (b : a)$. Furthermore, each of the terms $a', b', c', \dots$ asserts, since the denial is assertoric, and, in formula (3.), is also part of the disjunctive statement $a' + b' + c' + \dots$. But a disjunction cannot have assertions as part. A proposition, or a statement, in my opinion, does not assert, but says or states. If MacColl does indeed miss the Frege-point here, he is not the only logician to do so: as Kenny (1963, p. 228) observes, even the theory of judgement proposed by the main promulgator of the Frege-point, to wit Peter Geach in his *Mental Acts* (1957), violates the Frege-point and is really a theory of propositional content.

Matters become more obscure in

> Def. 3—The symbol :, which may be read "implies", asserts *that the statement following it must be true, provided the statement preceding it be true.*

Thus, the expression $a : b$ may be read "a implies b," or "If a is true, b must be true," or "Whenever a is true, b is also true".

Expressions of the form $a : b \dots$ (involving the symbol :) are called *Implications* or *Conditional Statements*. The statement to the left of the sign : is called the *Antecedent*, and the statement to the right of the sign : is called the *Consequent*.

How should *statement* be understood in definition 3? The obvious alternative is that the statements of MacColl are propositions. This works well with the three readings offered:

$$\text{That } S \text{ implies that } T;$$
$$\text{If that } S \text{ is true, that } T \text{ must be true;}$$
$$\text{Whenever that } S \text{ is true, that } T \text{ is true as well;}$$

all make grammatical sense (even though they will read more pleasantly after an application of transformation (8)).

However, when they are taken in the sense of proposition, according to the third reading "Whenever ...", the implication A implies B is true, when B is a logical consequence of A, where the latter notion is defined in the style of Bolzano and Tarski. That is, not only must the consequence $[A \Rightarrow B]$ hold, it has to hold logically in all cases. The second reading ('must be true'), on the other hand, which is what one would start with in explaining the validity of inference, seems to turn implication into an "entailment-connective" between propositions corresponding to the rendering of Bolzano's *Abfolge* that was suggested above.

In definition 4 (1880, p. 51) statement-identity is explained as implication (in the sense of MacColl, that is) in both directions; on the reading I have offered, this turns out to be the same theory as that offered by Wittgenstein in the *Tractatus*, namely that which identifies logically equivalent propositions. According to MacColl

$$a : b = (a = ab).$$

In modern terminology this would be:

$$A \models B \text{ is logically equivalent to the logical truth of } A \equiv A \,\&\, B.$$

From the point of view of standard modern logic, be it classical or constructive,

$$A \supset B \text{ is logically equivalent to } A \equiv A \,\&\, B.$$

This, however, is not what MacColl gets. His result is weaker owing to his very strong reading of implication.

A further difficulty lies in his terminology: both *Implication* and *Conditional Statement* can be used for symbols of the form $A : B$. (I prefer to use capital statement letters.) But ... implies ... only takes (my) propositions, whereas If ..., then ... only accepts (my) statements. Accordingly, the symbol : seems to do double duty, both for my implication $\supset$ and for my consequence $\Rightarrow$. In fact, as far as terminology is concerned, the sign : is also used as a symbol for inference:

=, the symbol of equivalence, and ... :, the symbol of *inference*, or implication. (MacColl 1880, p. 53, my emphasis)

So the statements of MacColl cover both my propositions and my statements. In fact, MacColl's statements serve as the minimal constituents of arguments, as we saw in the quote offered above. But in an argument assertions occur; otherwise no argumentative power is present. Accordingly, MacColl seems to use statements also as my asserted statements, or judgements made. If MacColl is guilty of this triple conflation, he is not alone: Frege took Peano to task for overburdening his $\supset$-sign with four or even five meanings, whereas the *consequentiae* of Scholastic logic had to allow for four different readings: implication, consequence, inference and causal grounding.

MacColl's definition (1880, p. 55) of the symbol $\therefore$, therefore, takes the form

$$A \therefore B = A \times (A : B).$$

But this again seems to ignore the Frege-point: in the locution

$$A. \text{ Therefore } B$$

the statement A carries assertoric force and cannot be put in antecedent position, whereas the right-hand side of MacColl's equation does not suffer from these liabilities.

Again, my insisting upon this letter of the logical law might be niggardly and uncharitable. MacColl makes the clear observation:

The statement $A \therefore B$ is stronger than the conditional statement $A : B$ and implies the latter. The former asserts that B is true *because* [emphasis added] A is true; the latter asserts that B is true *provided* that A be true. (MacColl 1880, p. 55)

The conditional statement does not, of its own, assert anything, but it can be asserted. An assertion made by means of it does assert that B

is true, provided that A is true (where A and B are propositions in my sense).

If I am right in the above, MacColl *does* ignore a number of basic distinctions in the foundations of logic, in particular concerning the differences between the implication(al proposition)

$$\text{that } R \supset \text{that } S,$$

the conditional statement

that S is true, on condition that it is true that R,

the consequence from that R to that S

$$\text{that } R \Rightarrow \text{that } S,$$

and the inference

that R is true. Therefore: that S is true.[10]

However, at this stage in the tradition of logic, almost everybody does so. Frege, certainly, had seen the Frege-point, but the difference between proposition and statement he did not have, and, in particular his *Begriffsschrift*-system suffers from this on a number of scores.[11]

In the examples I have considered from MacColl, invariably he hit upon something interesting or true, and sometimes both; there is more to be found, especially concerning the logic of epistemic notions (rather than epistemic logic) in the later instalments. Hugh MacColl was a pioneer and it is greatly to his credit to have pinpointed so surely such a wealth of crucial logical notions and issues.

[10] This was still the case in 1906a, where, in particular, the treatment of *therefore* in §§ 76–78, at pp. 80–83, continues to beg the Frege point.

[11] Note added in proof: The referee rightly observed that my stern reading of MacColl might not do him justice at this point; these conflations seem to have been at least partly resolved in later works, for instance, the 1906 book version of his symbolic reasonings. In that work, the statements are clearly best read as propositions. Furthermore, MacColl claims it to be an advantage of his system, rather than a serious drawback, that it allows for many readings; see his letter to Russell, May 15, 1905: "[the] enormous superiority of my system is due in great measure to the very principle which you find so defective, namely the principle of leaving to context everything in the reasoning or symbolical operations which it is not absolutely necessary to express." (I am indebted to Prof. Astroh for putting the letters of MacColl to Russell at my disposal.)

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