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# WAYS OF UNDERSTANDING HUGH MACCOLL'S CONCEPT OF *SYMBOLIC EXISTENCE*

Hugh MacColl (1837–1909) proposed, in several papers, a non-standard way of understanding the ontology underlying what we today call quantified propositions. His ideas, mixed with reflections about the use of arbitrary objects, were not greatly successful and were ruthlessly criticised by Bertrand Russell especially. The aim of this paper is to show that a thorough reading of MacColl's general understanding of *symbolic existence*, a concept which is connected with his view of traditional *hypotheticals*, elucidates his proposals on the role of ontology in logics. The interpretation of MacColl's concept of *symbolic existence* put forward in this paper and embedded in a dialogical system of free logic can be expressed in a nutshell: in an argumentation, it sometimes makes sense to restrict the use and introduction of singular terms in the context of quantification to a formal use of those terms. That is, the Proponent is allowed to use a constant iff this constant has been explicitly conceded by the Opponent. The paper also offers a second way of reconstructing MacColl's ideas on contradictory objects by means of combining the concept of formal use of constants in free logics and that of the formal use of elementary negations in paraconsistent logics.

## 1. Introduction

### *1.1. Aims of the paper*

Hugh MacColl (1837–1909) proposed, in several papers, a non-standard way of understanding the ontology underlying what we today call quantified propositions. His ideas, mixed with reflections about the use of arbitrary objects, were not greatly successful and were ruthlessly criticised by Bertrand Russell especially.

The aim of this paper is to show that a thorough reading of MacColl's general understanding of *symbolic existence*, a concept which is strongly connected with his view of *hypotheticals*, elucidates

*Nordic Journal of Philosophical Logic*, Vol. 3, No. 1, pp. 35–58.

MacColl's proposals on the role of ontology in logics. I will make an abstraction of MacColl's use of arbitrary objects by replacing them with quantifiers and will also make brief comments on the connections he establishes between symbolic existence and the formulation of a strong conditional. This move centres the discussion on the main idea, although I concede it may also bend MacColl's own argumentation style in some way.

### *1.2. Symbolic reasoning and the problematic modality of hypotheticals*

MacColl's formulation of traditional syllogistic is part of a general framework where rules of logic are considered as rules for *hypotheticals*. According to MacColl, Boole's logical equations for hypotheticals should be replaced by a system of equivalent propositions including disjunctions and conditionals, which reflects the natural semantics of traditional hypothetical forms. This natural semantics is best described by saying that a hypothetical form expresses (1) a necessary connection between the two parts of the hypothetical for the conditional and (2) some doubt on the part of the user of a given hypothetical as to the actual truth, in a given instance, of the pair of statements which compose this connection. The formal translation of the necessary connection between the two parts of the hypothetical in conditional form led MacColl to formulate a strong concept of implication, which in his early work was defined as relevant and connexive and in his later work as a strict implication. The translation of the problematic modality of hypotheticals was achieved through the distinction between formal and non-formal (or material) truth, which MacColl misses in Boole's use of the symbol '1': According to MacColl a complex formula expresses a hypothetical proposition if the subformulae occurring in it are stated hypothetically. The subformulae are said to be stated hypothetically if the truth-value of the complex formula is independent of the actual truth of these subformulae:

> The premisses $A : B$ and $B : C$ [i.e. $A \to B$ and $B \to C$—S.R.] of the latter [implication: $\{(A \to B) \wedge (B \to C)\} \to (A \to C)$—S.R.] are hypothetical concepts of the mind—concepts which may be true or false (as may also the conclusion), without in the least invalidating the formula (MacColl 1902, p. 368).

Actually, there is some ambiguity in MacColl's use of the word 'hypothetical'. Instead of describing hypotheticals as having subformulae with a problematic modality, he speaks, as already mentioned, of suformulae *stated hypothetically*. In general, we can say that when MacColl wishes to stress the problematic modality of propositions he calls them *statements*:

Def. 6.—Statements represented by letters or any other arbitrary symbols, to which we attach only a *temporary* meaning, are usually statements whose truth or falsehood may be considered an open question, like the statements of witnesses in a court of justice (MacColl 1880, p. 47).

MacColl's use of the word 'statement' is unfortunately not always consistent, but as I have argued elsewhere the concept of truth-determining propositional variables can be fruitfully applied instead (see Rahman 1998). This concept follows the lines of MacColl's main purposes and provides a good basis for reconstructing his reflections on hypotheticals. Thus I will say that a set of (occurrences of) propositional variables is *truth-determining* for a proposition $A$ iff the truth value of $A$ may be determined as true or false on all assignments of true or false to the set. I will say further that a propositional variable occurring in $A$ is *redundant* iff there is a truth-determining set for $A$ that does not contain this propositional variable. Thus, clearly, the set $\{p\}$ is not truth-determining for $p \rightarrow q$, but the set $\{p_1, p_2\}$ is truth-determining for $p \rightarrow p$. In this way we can reconstruct MacColl's use of elementary statements, that is, elementary propositions with a problematic mood, as truth-determining propositional variables, regarded as truth-determining independently of their actual truth-value—only their possible truth-values should be considered. By these means it is even possible to link the problematic modality of hypotheticals with the strong connection required for them, namely: A proposition $A$ is *strongly connected* iff no redundant propositional variable (nor any of the occurrences of a propositional variable) occurs in $A$.[1]

Now all this allows the ideas behind MacColl's general framework to be expressed in the following way: The elementary expressions of the logical language are truth-determining propositional variables. Propositions in which the actual truth-values of their propositional variables are known are called categoricals. Hypotheticals in which the actual truth-value of their propositional variables cannot modify the truth of the propositions in which they occur are formally valid. Symbolic reasoning is to reason under two conditions, namely: 1. Only those propositions are allowed in which no non-truth-determining propositional variables occur; 2. Propositional variables are used independently of their actual truth. The first condition yields a system with a strong conditional; the second condition, which reflects the problematic modality of traditional hypotheticals, commits itself to a formal use of propositional variables. In other words, to reason symbolically means to reason with hypotheticals.

[1]See details in Rahman 1997a, 1998 and Rahman and Rückert 1998, 1999a.

But what does it mean to use propositional variables formally? The dialogical approach to logic, which will be introduced in detail in the next section, has a very appealing answer to this question and offers a consistent way for understanding MacColl's reflections on the problematic modality of hypotheticals: Let an argumentation be given in which someone, the Proponent, states a thesis, and someone else, the Opponent, rejects it. In the course of the argumentation the use of the propositional variables is said to be used formally iff 1. Propositional variables cannot be attacked; 2. The Proponent may use a propositional variable in a move iff the Opponent has already stated the same statement before—that is, instead of committing himself to the empirical defence of a given atomic proposition $a$, the Proponent chooses the following way of justifying his use of $a$: "If you (the Opponent) concede that $a$ holds, so do I" (observe that, because of the difference between game and strategy levels in dialogical logic, the winning of a dialogue with help of the formal rule does not necessarily yield the validity of the formula involved—see 2.2).

Hugh MacColl tried to build a system for quantified propositions which should inherit this general framework (see Rahman 1997a, II). MacColl thought that this implied not only creating a system of first-order logic with a strong conditional but also the formulation of a system where the use of propositions stating facts about the elements of the corresponding universe of discourse commits one only to a symbolic existence of the objects introduced by these propositions. That is, MacColl tried to formulate a logic where even the use of constants assumes a problematic modality:

Let $e_1, e_2, e_3$, etc. (up to any number of individuals mentioned in our argument or investigation) denote our universe of *real existences*. Let $0_1, 0_2, 0_3$, etc., denote our universe of *non-existences*, that is to say, of unrealities, such as *centaurs*, *nectar*, *ambrosia*, *fairies*, with self-contradictions, such as *round squares*, *square circles*, *flat spheres*, etc., including, I fear, the non-Euclidean geometry of four dimensions and other hyperspatial geometries. Finally, let $S_1, S_2, S_3$, etc., denote our Symbolic Universe, or "Universe of Discourse," composed of all things real or unreal that are named or expressed by words or other symbols in our argument or investigation ...

When a class A belongs wholly to the universe $e$, or wholly to the class 0, we may call it a pure class ...

We may sum up briefly as follows: Firstly, when any symbol A denotes an *individual*; then, any intelligible statement $\phi(A)$, containing the symbol A, implies that the individual represented by A has a *symbolic* existence; but whether the statement $\phi(A)$ implies that the individual represented by A has a *real* existence depends upon the context. Secondly, when any symbol A denotes a *class*, then, any intelligible statement $\phi(A)$ containing the symbol A implies that the whole class A has a *symbolic* existence; but whether the statement $\phi(A)$ implies that the class A is *wholly real*, or *wholly unreal*, or *partly real and partly unreal*, depends upon the context. (MacColl 1905b, pp. 74–77; see also MacColl 1905a,b and MacColl 1906, pp. 76–77).

But how can i) a symbolic use of constants, ii) classifications in such a symbolic universe of discourse, and iii) propositions about flat spheres and round squares be introduced in formal logic?

All these questions can be answered in the context of the dialogical approach to free logic developed recently (Rahman et al. 1999) in a way which is congenial with MacColl's proposal of formulating a first-order logic which reflects the problematic modality of propositional logic. In a nutshell: in an argumentation, it sometimes makes sense to restrict the use and introduction of singular terms in the context of quantification to a formal (or *symbolic*) use of those terms. That is, the Proponent is allowed to use a constant iff this constant has been explicitly conceded by the Opponent. The symbolic use of constants amounts to allowing the use of these constants under the sole restriction that they name an individual: their ontological characterisation besides individuality does not play any role in logics. This paper also offers a second way of reconstructing MacColl's ideas on contradictory objects by means of combining the concept of formal use of constants in free logics and that of the formal use of elementary negations in paraconsistent logics.

In the next section I do not go into the details of free logics based on reference. Instead, I show how the dialogical approach to free logic can capture the ideas behind MacColl's concept of symbolic existence.

## 2. Symbolic Existence and the Dialogical Approach to Free Logic

### *2.1. The dialogical approach to free logic*

Dialogical logic, suggested by Paul Lorenzen in 1958 and developed by Kuno Lorenz in several papers from 1961 onwards,[2] was introduced as a pragmatical semantics for both classical and intuitionistic logic.

The dialogical approach studies logic as an inherently pragmatic notion with the help of an overtly externalised argumentation formulated as a *dialogue* between two parties taking up the roles of an *Opponent* (O in the following) and a *Proponent* (P) of the issue at stake, called the principal *thesis* of the dialogue. P has to try to defend the thesis against all possible allowed criticism (*attacks*) by O, thereby being allowed to use statements that O may have made at the outset of the dialogue. The thesis $A$ is logically valid if and only if P can succeed in

[2] Lorenzen and Lorenz 1978. Further work has been done by Rahman (1993).

defending $A$ against all possible allowed criticism by the Opponent. In the jargon of game theory: P has a *winning strategy* for $A$.

The philosophical point of dialogical logic is that this approach does not understand semantics as mapping names and relationships into the real world to obtain an abstract counterpart of it, but as acting upon them in a particular way.

I will now describe an intuitionistic and a classical version of a very basic system called DFL (dialogical free logic) introduced in Rahman et al. 1999. Since the principal aim of the paper is the elucidation of MacColl's concept of symbolic existence, I will not introduce a system which contemplates strongly connected propositions. For such a system, cf. Rahman 1997a, 1998, Rahman and Rückert 1998, 1999a.

Suppose the elements of first-order language are given with small letters $(a, b, c, \dots)$ for elementary formulae, capital italic letters for formulae that might be complex $(A, B, C, \dots)$, capital sans serif letters $(\mathsf{A}, \mathsf{B}, \mathsf{C}, \dots)$ for predicators and constants $\tau_i$. A dialogue is a sequence of labelled formulae of this first-order language that are stated by either P or O.[3] The label of a formula describes its role in the dialogue, whether it is an aggressive or a defensive act. An *attack* is labelled with $?_{n/\dots}$, while $!_{n/\dots}$ tags a defence. ($n$ is the number of the formula the attack or defence reacts to, and the dots are sometimes completed with more information. The use of indices on labels will be made clear in the following). In dialogical logic the meaning in use of the logical particles is given by two types of rules which determine their *local* (*particle rules*) and their *global* (*structural rules*) meaning. The particle rules specify for each particle a pair of moves consisting of an attack and (if possible) the corresponding defence. Each such pair is called a *round*. An attack *opens* a round, which in turn is *closed* by a defence if possible. Before presenting a dialogical system DFL for free logics, we need the following definition.

DEFINITION 1. *A constant* $\tau$ *is said to be* introduced by X *if (1)* X *states a formula* $A[\tau/x]$ *to defend* $\bigvee_x A$ *or (2)* X *attacks a formula* $\bigwedge_x A$ *with* $?_{n/\tau}$*, and* $\tau$ *has not been used in the same way before. Moreover, an atomic formula is said to be introduced by* X *if it is stated by* X *and has not been stated before.*

DFL is closely related to Lorenz's standard dialogues for both intuitionistic and classical logic. The particle rules are identical, and the sets of structural rules differ in only one point, namely when determining the way constants are dealt with. Before presenting the formal definition of DFL, we should take a look at a simple propositional dialogue as an example of notational conventions:

[3] Sometimes I use X and Y to denote P and O with X $\neq$ Y.

| Opponent | | Proponent | |
|---|---|---|---|
| | | $a \wedge b \rightarrow a$ | (0) |
| (1) | $?_0 a \wedge b$ | $!_1 a$ | (4) |
| (3) | $!_2 a$ | $?_{1/\text{left}}$ | (2) |
| | | *The Proponent wins* | |

Formulae are labelled in (chronological) order of appearance. They are not listed in the order of utterance, but in such a way that every defence appears on the same level as the corresponding attack. Informally, the argument goes like this:

P: "If $a$ and $b$, then $a$."
O: "Given $a$ and $b$, show me that $a$ holds."
P: "If you assume $a$ and $b$, you should be able to show me that both hold. Thus show me that the left part holds."
O: "OK, $a$."
P: "If you can say that $a$ holds, so can I."
O runs out of arguments, P wins.

Definition 2. Particle Rules.

| | Attack | Defence |
|---|---|---|
| $\neg A$ | $?_n A$ | $\otimes$ *(The symbol '$\otimes$' indicates that no defence, but only counterattack is allowed)* |
| $A \wedge B$ | $?_{n/\text{left}}$ / $?_{n/\text{right}}$ *(The attacker chooses)* | $!_m A$ / $!_m B$ |
| $A \vee B$ | $?_n$ | $!_m A$ / $!_m B$ *(The defender chooses)* |
| $A \rightarrow B$ | $?_n A$ | $!_m B$ |
| $\bigwedge_x A$ | $?_{n/\tau}$ *(The attacker chooses)* | $!_m A[\tau/x]$ |
| $\bigvee_x A$ | $?_n$ | $!_m A[\tau/x]$ *(The defender chooses)* |

The first row contains the form of the formula in question, the second row possible attacks against this formula, and the last one possible defences against those attacks. (The symbol "⊗" indicates that no defence is possible.) Note that $?_{n/\ldots}$ is a move—more precisely it is an attack—and not a formula. Thus if one partner in the dialogue states a conjunction, the other may initiate the attack by asking either for the left side of the conjunction ("show me that the left side of the conjunction holds", or $?_{n/\text{left}}$ for short) or the right one ("show me that the right side of the conjunction holds", or $?_{n/\text{right}}$). If one partner in the dialogue states a disjunction, the other may initiate the attack by asking to be shown *any* side of the disjunction ($?_n$). As already mentioned, the number in the index denotes the formula the attack refers to. The notation of defences is used in analogy to that of attacks. Rules for quantifiers work similarly.

Next, we fix the way formulae are sequenced to form dialogues with a set of structural rules (orig. *Rahmenregeln*).

(DFL0). *Formulae are alternately uttered by* P *and* O*. The* initial formula *is uttered by* P*. It does not have a label, but provides the topic of argument. Every formula below the initial formula is either an attack or a defence against an earlier formula of the other player.*

(DFL1). *Both* P *and* O *may only make moves that change the situation.*[4]

(DFL2). FORMAL RULE FOR ATOMIC FORMULAE. P *may not introduce atomic formulae: any atomic formula must be stated by* O *first.*

(DFL3). FORMAL RULE FOR CONSTANTS. *Only* O *may introduce constants.*

(DFL4). WINNING RULE. X *wins iff it is* Y*'s turn but he cannot move (either attack or defend).*

($\text{DFL}_\text{I}5$). INTUITIONISTIC RULE. *In any move, each player may attack a (complex) formula asserted by his partner or he may defend himself against* the last not already defended *attack. Only the latest open attack may be answered. If it is* X*'s turn at position $n$ and there are two open attacks $m, l$ such that $m < l < n$, then* X *may not defend against $m$.*

DFL is an intuitionistic as well as a classical semantics. To obtain the classical version simply replace ($\text{DFL}_\text{I}5$) by the following rule:

($\text{DFL}_\text{C}5$). CLASSICAL RULE. *In any move, each player may attack a (complex) formula asserted by his partner or he may defend himself against* any attack *(including those which have already been defended).*

[4] This rule replaces Lorenz's *Angriffsschranken*, but this point still remains to be made clear on a formal basis. The idea is that a new situation is defined when O provides an atomic formula which can be used by the Proponent.

If we need to specify (explicitly) which system is meant, we write $\mathrm{DFL_I}$ or $\mathrm{DFL_C}$ instead of DFL.

The crucial rule that makes DFL behave like a free logic is (DFL3).

To see the difference between standard and free dialogues (those with and those without (DFL3)), consider another example. Without (DFL3), we would obtain the following dialogue proving that if nothing is a vampire, Nosferatu is no vampire:

| | Opponent | Proponent | |
|---|---|---|---|
| | | $\bigwedge_x \neg \mathsf{A}_x \to \neg \mathsf{A}_\tau$ | (0) |
| (1) | $?_0 \bigwedge_x \neg \mathsf{A}_x$ | $!_1 \neg \mathsf{A}_\tau$ | (2) |
| (3) | $?_2 \mathsf{A}_\tau$ | $\otimes$ | |
| (5) | $!_4 \neg \mathsf{A}_\tau$ | $?_{1/\tau}$ | (4) |
| | $\otimes$ | $?_5 \mathsf{A}_\tau$ | (6) |
| | | *The Proponent wins* | |

If we play the same dialogue again in DFL, things look different:

| | Opponent | Proponent | |
|---|---|---|---|
| | | $\bigwedge_x \neg \mathsf{A}_x \to \neg \mathsf{A}_\tau$ | (0) |
| (1) | $?_0 \bigwedge_x \neg \mathsf{A}_x$ | $!_1 \neg \mathsf{A}_\tau$ | (2) |
| (3) | $?_2 \mathsf{A}_\tau$ | $\otimes$ | |
| | *The Opponent wins* | | |

We observe that P runs out of arguments. He cannot attack (1) any more, because not a single constant has been introduced so far, and he may not introduce one on its own. Neither can he defend himself against the atomic formula in (3) due to the particle rule for negation.

It is obvious that the (Proponent's) thesis $A_\tau \to \bigvee_x A_x$ cannot be won. This shows that the Opponent may state a proposition about a fictive entity without committing himself to its existence. MacColl's reflections on non-existence amount to this analysis of $A_\tau \to \bigvee_x A_x$.[5]

*2.2. Winning strategies and dialogical tableaux for DFL*

As already mentioned, validity is defined in dialogical logic via winning strategies of P, i.e. the thesis $A$ is logically valid iff P can succeed in defending $A$ against all possible allowed criticism by O. In this case, P has a *winning strategy* for $A$. It should be clear that the formal rule

[5] M. Astroh's thorough discussion of MacColl's conception of existence (Astroh 1996, pp. 1399–1401) amounts to the failure of this thesis in DFL.

which elucidates MacColl's understanding of the problematic modality of hypotheticals does not necessarily imply that winning a dialogue with the help of this rule yields the validity of the formula involved: The Proponent may win a dialogue, even formally, because the Opponent did not play the best moves. Validity, on the other hand, forces the consideration of all possibilities available. A systematic description of the winning strategies available can be obtained from the following considerations:

*If* P *shall win against any choice of* O*, we will have to consider two main different situations, namely the dialogical situations in which* O *has stated a complex formula and those in which* P *has stated a complex formula. We call these main situations the* O*-cases and the* P*-cases, respectively.*

In both of these situations another distinction has to be examined:

1. P wins by *choosing* an attack in the O-cases or a defence in the P-cases, iff he can win *at least one* of the dialogues he has chosen.

2. When O can *choose* a defence in the O-cases or an attack in the P-cases, P can win iff he can win *all of the* dialogues O can choose.

The closing rules for dialogical tableaux are the usual ones: a branch is closed iff it contains two copies of the same formula, one stated by O and the other one by P. A tree is closed iff each branch is closed.

For the intuitionistic tableaux, the structural rule about the restriction on defences has to be considered. The idea is quite simple: the tableaux system allows all the possible defences (even the atomic ones) to be written down, but as soon as determinate formulae (negations, conditionals, universal quantifiers) of P are attacked all others will be deleted. Those formulae which compel the rest of P's formulae to be deleted will be indicated with the expression "$O_{[O]}$" (or "$P_{[O]}$"), which reads *save* O*'s formulae and delete all of* P*'s formulae stated before.*

To obtain a tableaux system for DFL from those described above, add the following restriction to the closing rules and recall the rule (DFL3) for constants:

DFL-RESTRICTION. *Check that for every step in which* P *chooses a constant (i.e. for every* P*-attack on a universally quantified* O*-formula and for every* P*-defence of an existentially quantified* P*-formula) this constant has been already introduced by* O *(by means of an* O*-attack on a universally quantified* P*-formula or a defence of an existentially quantified* O*-formula).*

This restriction can be technically implemented by a device which provides a label (namely a star) for each constant introduced by O. Thus, the DFL-restriction can be simplified in the following way:

DFL-RESTRICTION WITH LABELS. *Check that for every step in which* P *chooses a constant this constant has already been there, labelled with a star.*

All these considerations can be expressed by means of the tableaux systems for classical and intuitionistic DFL.[6]

CLASSICAL TABLEAUX FOR DFL.

| (O)-Cases | (P)-Cases |
|---|---|
| $\dfrac{(\mathrm{O})A \wedge B}{\langle(\mathrm{P})_?\rangle(\mathrm{O})A \mid \langle(\mathrm{P})_?\rangle(\mathrm{O})B}$ | $\dfrac{(\mathrm{P})A \wedge B}{\begin{matrix}\langle(\mathrm{O})_?\rangle(\mathrm{P})A \\ (\langle(\mathrm{O})_?\rangle(\mathrm{P})B)\end{matrix}}$ |
| $\dfrac{(\mathrm{O})A \wedge B}{\begin{matrix}\langle(\mathrm{P})_{?\mathrm{left}}\rangle(\mathrm{O})A \\ (\langle(\mathrm{P})_{?\mathrm{right}}\rangle(\mathrm{O})B)\end{matrix}}$ | $\dfrac{(\mathrm{P})A \wedge B}{\langle(\mathrm{O})_{?\mathrm{left}}\rangle(\mathrm{P})A \mid \langle(\mathrm{O})_{?\mathrm{right}}\rangle(\mathrm{P})B}$ |
| $\dfrac{(\mathrm{O})A \to B}{(\mathrm{P})A \ldots \mid \langle(\mathrm{P})A\rangle(\mathrm{O})B}$ | $\dfrac{(\mathrm{P})A \to B}{\begin{matrix}(\mathrm{O})A \\ (\mathrm{P})B\end{matrix}}$ |
| $\dfrac{(\mathrm{O})\neg A}{(\mathrm{P})A, \otimes}$ | $\dfrac{(\mathrm{P})\neg A}{(\mathrm{O})A, \otimes}$ |
| $\dfrac{(\mathrm{O})\bigwedge_x A}{\langle(\mathrm{P})_{?\tau}\rangle(\mathrm{O})A_{[\tau^*/x]}}$ *$\tau$ has been labelled with a star before* | $\dfrac{(\mathrm{P})\bigwedge_x A}{\langle(\mathrm{O})_{?\tau^*}\rangle(\mathrm{P})A_{[\tau^*/x]\tau}}$ *$\tau$ is new* |
| $\dfrac{(\mathrm{O})\bigvee_x A}{\langle(\mathrm{P})_?\rangle(\mathrm{O})A_{[\tau^*/x]}}$ *$\tau$ is new* | $\dfrac{(\mathrm{P})\bigvee_x A}{\langle(\mathrm{O})_?\rangle(\mathrm{P})A_{[\tau/x]}}$ *$\tau$ has been labelled with a star before* |

[6] See details on how to build tableaux systems from dialogues in Rahman 1993 and Rahman and Rückert 1997. The use of these tableaux systems follows the very well-known analytic trees of Raymund Smullyan (1968).

*The closing rules are the usual ones. Observe that the formulae below the line represent pairs of attack-defence moves. In other words, they represent rounds.*

*Note that the expressions between the symbols '⟨' and '⟩', such as* $\langle(\mathrm{P})_?\rangle$, $\langle(\mathrm{O})_?\rangle$ *or* $\langle(\mathrm{P})A\rangle$ *are moves—more precisely they are attacks—but not statements.*

Intuitionistic Tableaux for DFL.

(O)-Cases

$$\frac{(\mathrm{O})A \wedge B}{\langle(\mathrm{P})_?\rangle(\mathrm{O})A \mid \langle(\mathrm{P})_?\rangle(\mathrm{O})B}$$

$$\frac{(\mathrm{O})A \wedge B}{\begin{array}{c}\langle(\mathrm{P})_{?\mathrm{left}}\rangle(\mathrm{O})A \\ (\langle(\mathrm{P})_{?\mathrm{right}}\rangle(\mathrm{O})B)\end{array}}$$

$$\frac{(\mathrm{O})A \to B}{(\mathrm{P})A \ldots \mid \langle(\mathrm{P})A\rangle(\mathrm{O})B}$$

$$\frac{(\mathrm{O})\neg A}{(\mathrm{P})A, \otimes}$$

$$\frac{(\mathrm{O})\bigwedge_x A}{\langle(\mathrm{P})_{?\tau}\rangle(\mathrm{O})A_{[\tau^*/x]}}$$

*τ has been labelled with a star before*

$$\frac{(\mathrm{O})\bigvee_x A}{\langle(\mathrm{P})_?\rangle(\mathrm{O})A_{[\tau^*/x]}}$$

*τ is new*

(P)-Cases

$$\frac{(\mathrm{P})A \wedge B}{\begin{array}{c}\langle(\mathrm{O})_?\rangle(\mathrm{P})A \\ (\langle(\mathrm{O})_?\rangle(\mathrm{P})B)\end{array}}$$

$$\frac{(\mathrm{P})A \wedge B}{\langle(\mathrm{O})_{?\mathrm{left}}\rangle(\mathrm{P})A \mid \langle(\mathrm{O})_{?\mathrm{right}}\rangle(\mathrm{P})B}$$

$$\frac{(\mathrm{P})A \to B}{\begin{array}{c}(\mathrm{O})_{[\mathrm{O}]}A \\ (\mathrm{P})B\end{array}}$$

$$\frac{(\mathrm{P})\neg A}{(\mathrm{O})_{[\mathrm{O}]}A, \otimes}$$

$$\frac{(\mathrm{P})\bigwedge_x A}{\langle(\mathrm{O})_{?\tau^*}\rangle(\mathrm{P})_{[\mathrm{O}]}A_{[\tau^*/x]\tau}}$$

*τ is new*

$$\frac{(\mathrm{P})\bigvee_x A}{\langle(\mathrm{O})_?\rangle(\mathrm{P})A_{[\tau/x]}}$$

*τ has been labelled with a star before*

Let us look at two examples, namely one for classical DFL and one for intuitionistic DFL:

| | |
|---|---|
| (P) | $\bigwedge_x \neg \mathsf{A}_x \rightarrow \neg \mathsf{A}_\tau$ |
| (O) | $\bigwedge_x \neg \mathsf{A}_x$ |
| (P) | $\neg \mathsf{A}_t$ |
| (O) | $\mathsf{A}_t$ |

The tableau remains open as P cannot choose $\tau$ to attack the universal quantifier of O. The following intuitionistic tableau is slightly more complex:

| | | |
|---|---|---|
| | ~~(P)~~ | ~~$\bigwedge_x \mathsf{A}_x \rightarrow \neg \bigvee_x \neg \mathsf{A}_x$~~ |
| | $(\mathrm{O})_{[\mathrm{O}]}$ | $\bigwedge_x \mathsf{A}_x$ |
| | ~~(P)~~ | ~~$\neg \bigvee_x \neg \mathsf{A}_x$~~ |
| | $(\mathrm{O})_{[\mathrm{O}]}$ | $\bigvee_x \neg \mathsf{A}_x$ |
| $\langle (\mathrm{P})_{?} \rangle$ | (O) | $\neg \mathsf{A}_{t^*}$ |
| $\langle (\mathrm{P})_{?\tau} \rangle$ | (O) | $\mathsf{A}_t$ |
| | (P) | $\mathsf{A}_t$ |

*The tree closes.*

### *2.3. Many quantifiers and sorts of objects in the symbolic universe: The systems DFL$^n$ and DFL$^{\langle n \rangle}$*

Consider the situation expressed by the following proposition:

> *The novel contains a passage in which Sherlock Holmes dreams that he shot Dr. Watson.*

There is an underlying reality that the novel is part of, the outer reality of the story told in the novel and an even further outer reality of the dream of the protagonist. To distinguish between the reality of Conan Doyle writing stories, Holmes's reality and the reality of Holmes' dream, we need three pairs of quantifiers expressing the three sorts of reality (or fiction), for which in a first step we do not need to assume that they introduce an order of levels of fiction (or reality). Actually MacColl, as can be read in the text quoted in 1.2, formulated a system in which different sorts of elements of the symbolic universe are considered. Now, if the motivation of introducing a formal use of constants (or in MacColl's words a symbolic universe) is, as already mentioned, an ontologically neutral treatment of these constants, it is not very clear why levels of reality should be considered at all. Narahari Rao, for example, thinks that such a graduation is incompatible with the very idea of a symbolic universe (see Rao 1999). Formally, the introduction of sorts of elements is very simple: Think of the pair of quantifiers of

DFL as having the upper index 0 and add new pairs of quantifiers with higher indices, as many as we need to express every sort of reality (or fiction) that could possibly appear. We call the dialogical logic thus derived $\mathrm{DFL}^{\mathrm{n}}$. The new particle rules to be added to DFL are:

| | Attack | Defence |
|---|---|---|
| $\bigwedge_x^i A$ | $?_{n/\tau}$ (The attacker chooses) | $!_m A[\tau/x]$ |
| $\bigvee_x^i A$ | $?_n$ | $!_{m/\tau} A[\tau/x]$ (The defender chooses) |

The extended set of quantifiers requires a new notion of introduction.

Definition 3. *A constant $\tau$ is said to be* introduced as belonging to the sort *$i$ iff it is used to attack a universal quantifier of sort $i$ or to defend an existential quantifier of sort $i$ and has not been used in the same way before.*

I adapt (DFL3) to $\mathrm{DFL}^{\mathrm{n}}$:

($\mathrm{DFL}^{\mathrm{n}}$3). First extended formal rule for constants. *For each sort of quantification the following rule holds: constants may only be introduced by* O.

These formulations yield a logic containing an arbitrary number of disjoint pairs of quantifiers dealing with different sorts of reality and fiction—this logic contains also the standard (non-free quantifiers) $\exists$ and $\forall$ for which neither DFL3 nor $\mathrm{DFL}^{\mathrm{n}}$3 hold.

In some contexts, it might be useful to have a logic where these different realities are ordered in a hierarchy. We call the system that establishes this ordering $\mathrm{DFL}^{\langle\mathrm{n}\rangle}$; it results from modifying (DFL3) again:

($\mathrm{DFL}^{\langle\mathrm{n}\rangle}$3). Second extended formal rule for constants. P *may introduce a constant $\tau$ on a level $m$ iff* O *has introduced $\tau$ on some level $n$ with $n < m$ before.*

I leave as an exercise for the reader two examples. The first states that in $\mathrm{DFL}^{\mathrm{n}}$, whenever $A$ has an instance in the scope of one or another $\vee$-quantifier, it has an instance in the scope of $\exists$; the second makes use of the ordering in $\mathrm{DFL}^{\langle\mathrm{n}\rangle}$:

1. $(\bigvee_x^1 \mathsf{A}_x \vee \bigvee_x^2 \mathsf{A}_x) \rightarrow \exists_x \mathsf{A}_x$ (to be solved with $\mathrm{DFL}^{n}$)

2. $(\bigvee_x^1 \mathsf{A}_x \wedge \bigwedge_x^2 (\mathsf{A}_x \rightarrow \mathsf{B}_x)) \bigvee_x^1 \mathsf{B}_x$ (to be solved with $\mathrm{DFL}^{\langle n\rangle}$)

## 3. Symbolic Universe and Inconsistent Objects

*3.1. Paraconsistency*

MacColl's system contains inconsistent objects like round squares, flat spheres and so on. The logic described above can deal with these objects as elements of the symbolic universe. Another way of dealing with this situation is to understand arguments containing propositions about inconsistent objects as arguments in which inconsistent elementary propositions about given elements of the universe of discourse are allowed. That is, instead of allowing the use of constants which name inconsistent objects, you have arguments in which two contradictory elementary propositions are allowed—this way of thinking about inconsistent objects was proposed by Richard Routley (see Routley 1979) in his interpretation of Felix Meinong (for a brief exposition of the main ideas of Richard Routley's monumental work, see Manuel Bremer 1998). This requires a logic in which such contradictions are allowed. Such a logic was the aim of the founders of paraconsistent logic, namely the Polish logician Stanisl aw Jaśkowski (see Jaśkowski 1948) and the Brazilian logician Newton C. A. da Costa (see da Costa 1974).

The work of da Costa takes the assumption that contradictions can appear in a logical system without making this system trivial. Actually this leads to the standard definition of paraconsistent logics:

Definition 4. Paraconsistency. *Let us consider a theory $\mathcal{T}$ as a triple $\langle \mathcal{L}, \mathcal{A}, \mathcal{G} \rangle$, where $\mathcal{L}$ is a language, $\mathcal{A}$ is a set of propositions (closed formulae) of $\mathcal{L}$, called the axioms of $\mathcal{T}$, and $\mathcal{G}$ is the underlying logic of $\mathcal{T}$. We suppose that $\mathcal{L}$ has a negation symbol, and that, as usual, the theorems of $\mathcal{T}$ are derived from $\mathcal{A}$ by the rules of $\mathcal{G}$ (cf. da Costa et al. 1998, p. 46).*

*In such a context, $\mathcal{T}$ is said to be* inconsistent *if it has two theorems $A$ and $\neg A$, where $A$ is a formula of $\mathcal{L}$. $\mathcal{T}$ is called* trivial *if any formula of $\mathcal{L}$ is a theorem of $\mathcal{T}$. $\mathcal{T}$ is called* paraconsistent *if it can be inconsistent without being trivial. Equivalently $\mathcal{T}$ is paraconsistent if it is not the case that when $A$ and $\neg A$ hold in $\mathcal{T}$, any $B$ (from $\mathcal{L}$) also holds in $\mathcal{T}$.*

Thus, if $\mathcal{T}$ is a paraconsistent theory it is not the case that every formula of $\mathcal{L}$ and its negation are theorems of $\mathcal{T}$. Typically, in a paraconsistent theory $\mathcal{T}$, there are theorems whose negations are not theorems of $\mathcal{T}$. Nonetheless, there are formulae which are theorems of $\mathcal{T}$ and whose negations are also theorems (da Costa et al. 1998, p. 46).

Actually there are two main interpretations possible. The one, which I will call the *compelling interpretation*, based on a naive correspondence theory, stresses that paraconsistent theories are ontologically committed to inconsistent objects. The other, which I call the *permissive interpretation*, does not assume this ontological commitment of paraconsistent theories. The usual referential semantics for paraconsistent logics is not really compatible with the idea of a permissive interpretation of paraconsistency—the permissive interpretation of inconsistencies can be seen as another way of stating their symbolic existence. Rahman and Carnielli (1998) developed a dialogical approach to paraconsistency which yields several systems called *literal dialogues* (shorter: L-D) and takes its permissive non-referential interpretation seriously. I will adapt L-D to the purposes of the present paper.

*3.2. The dialogical approach to paraconsistent logic*

As already mentioned, MacColl's symbolic universe contains non-existent objects and (formally) existent ones. Non-existent objects are in my reconstruction those objects which are named by constants that have been *used*—i.e. which occur in a formula stated in a dialogue—without having been introduced (in the sense of DFL3) before. Now, contradictory objects are in MacColl's view to be included in the subuniverse of non-existent objects, and this is quite in the sense of a permissive interpretation of paraconsistency. Thus, I will provide the system(s) of free-logic DFL with a rule introducing paraconsistency—I call this rule the *negative literal rule* (DFL4)—but with the following caveat: *The* negative literal rule *applies only for formulae in which constants occur that have not been introduced in the sense of DFL3.*

(DFL4). Negative literal rule. *The Proponent is allowed to attack the negation of an atomic (propositional) statement (the so-called* negative literal*) if and only if the Opponent has attacked the* same *statement before.*

This structural rule can be considered analogous with the formal rule for positive literals. The idea behind this rule can be connected with MacColl's concept of symbolic existence in the following way: A contradiction between literals, say $a$ and $\neg a$, expresses that one proposition ascribes a predicator to a given object while the other denies that a predicator applies to this object. Now, if the Opponent is the one who introduces such a contradiction between literals, this contradiction can be seen as having a pure problematic modality, i.e. as being stated symbolically. This means that the Proponent—who has proposed $(a \wedge \neg a) \rightarrow \neg a$, for example—is also allowed to use the conceded symbolical contradiction $a \wedge \neg a$ (of the Opponent), stating himself for example $\neg a$. Expressed intuitively: "If you (the Opponent) concede that a flat sphere is not flat, so can I (the Proponent)". Now, suppose that the Opponent attacks $\neg a$ with $a$. This allows the Proponent to attack the corresponding negation (and no other) of the Opponent (i.e., "If you (the Opponent) attack my proposition that a flat sphere is not flat, so can I (the Proponent)").

When I want to distinguish between the intuitionistic and the classical version I write L-D$^{i}$ (for the intuitionistic version) and L-D$^{c}$ (for the classical version). To be precise, we should call these logical systems literal dialogues with classical structural rule and literal dialogues with intuitionistic structural rule, respectively. Actually, *strictu sensu* they are neither classical nor intuitionistic because neither in L-D$^{i}$ nor in L-D$^{c}$ are *ex falso sequitur quodlibet*, $(a \rightarrow b) \rightarrow ((a \rightarrow \neg b) \rightarrow \neg a)$, or $a \rightarrow \neg\neg a$ winnable.

In L-D the (from a paraconsistent point of view) dangerous formulae $(a \wedge \neg a) \rightarrow b$, $a \rightarrow (\neg a \rightarrow b)$ and $(a \rightarrow b) \rightarrow ((a \rightarrow \neg b) \rightarrow \neg a)$ are not valid. Let us see the corresponding literal dialogues in L-D$^{c}$ for the first and the last one:

| | Opponent | Proponent | |
|---|---|---|---|
| | | $(a \wedge \neg a) \rightarrow b$ | (0) |
| (1) | $?_0 a \wedge \neg a$ | | |
| (3) | $!_2 a$ | $?_{1/\text{left}}$ | (2) |
| (5) | $!_2 \neg a$ | $?_{1/\text{right}}$ | (4) |
| | *The Opponent wins* | | |

The Proponent loses because he is not allowed to attack the move (5) (see negative literal rule). In other words, the Opponent has stated the contradiction $a \wedge \neg a$ about an object, but this contradiction, being conceded as part of the symbolic reasoning in the argument, cannot be attacked by the Proponent until the Opponent starts an attack on the negative literal $\neg a$—an attack which in this case will not take place.

Similar considerations hold for $(a \rightarrow b) \rightarrow ((a \rightarrow \neg b) \rightarrow \neg a)$:

| | Opponent | Proponent | |
|---|---|---|---|
| | | $(a \rightarrow b) \rightarrow ((a \rightarrow \neg b) \rightarrow \neg a)$ | (0) |
| (1) | $?_0 a \rightarrow b$ | $!_1 (a \rightarrow \neg b) \rightarrow \neg a$ | (2) |
| (3) | $?_2 a \rightarrow \neg b$ | $!_3 \neg a$ | (4) |
| (5) | $?_4 a$ | $\otimes$ | |
| (7) | $!_6 b$ | $?_1 a$ | (6) |
| (9) | $!_8 \neg b$ | $?_3 a$ | (8) |
| | *The Opponent wins* | | |

The Proponent loses here because he cannot attack $\neg b$.

All classically valid formulae without negation are also valid in L-D$^c$. All intuitionistically valid formulae without negation are also valid in L-D$^i$. As in da Costa's system $C_1$ none of the following are valid in L-D$^c$:

| | |
|---|---|
| $(a \wedge \neg a) \rightarrow b$ | $(a \rightarrow (b \vee c)) \rightarrow ((a \wedge \neg b) \rightarrow c)$ |
| $(a \wedge \neg a) \rightarrow \neg b$ | $((a \rightarrow \neg a) \wedge (\neg a \rightarrow a)) \rightarrow \neg b$ |
| $\neg(a \wedge \neg a)$ | $((a \wedge b) \rightarrow c) \rightarrow ((a \wedge \neg c) \rightarrow \neg b)$ |
| $a \rightarrow \neg\neg a$ | $(a \rightarrow b) \vee (\neg a \rightarrow b)$ |
| $(a \rightarrow b) \rightarrow ((a \rightarrow \neg b) \rightarrow \neg a)$ | $((a \vee b) \wedge \neg a) \rightarrow b$ |
| $((a \rightarrow b) \wedge (a \rightarrow \neg b)) \rightarrow \neg a$ | $(a \vee b) \rightarrow (\neg a \rightarrow b)$ |
| $((\neg a \rightarrow b) \wedge (\neg a \rightarrow \neg b)) \rightarrow a$ | $(a \rightarrow b) \rightarrow (\neg b \rightarrow \neg a)$ |
| $\neg a \rightarrow (a \rightarrow b)$ | $(\neg a \vee \neg b) \rightarrow \neg(a \wedge b)$ |
| $\neg a \rightarrow (a \rightarrow \neg b)$ | $(\neg a \wedge \neg b) \rightarrow \neg(a \vee b)$ |
| $a \rightarrow (\neg a \rightarrow b)$ | $(\neg a \vee b) \rightarrow (a \rightarrow b)$ |
| $a \rightarrow (\neg a \rightarrow \neg b)$ | $(a \rightarrow b) \rightarrow \neg(a \wedge \neg b)$ |
| $((a \rightarrow \neg a) \wedge (\neg a \rightarrow a)) \rightarrow b$ | $\neg a \rightarrow ((a \vee b) \rightarrow b)$ |

In L-D$^i$ all the intuitionistically non-valid formulae have to be added to the list, for example:

| | |
|---|---|
| $\neg\neg A \rightarrow A$ | $A \vee (A \rightarrow B)$ |
| $A \vee \neg A$ | $A \vee ((A \vee B) \rightarrow B)$ |
| $((A \rightarrow B) \rightarrow A) \rightarrow A$ | $\neg(A \rightarrow B) \rightarrow A$ |

The extension of literal dialogues for propositional logic to first-order quantifiers is straightforward. To build *Quantified Literal Dialogues*, we have only to extend the structural negative literal rule to elementary statements of first-order logic. The way to do that is to generalise the rule for elementary statements:

DEFINITION 5. (GENERAL) NEGATIVE LITERAL RULE. *The Proponent is allowed to attack the negation of an elementary statement (i.e., the* negative literal*) if and only if the Opponent has attacked the* same *statement before.*

Let us look at an example:

| | Opponent | Proponent | |
|---|---|---|---|
| | | $\bigwedge_x((\mathsf{A}_x \wedge \neg\mathsf{A}_x) \rightarrow \mathsf{B}_x)$ | (0) |
| (1) | $?_{0/\tau}$ | $!_1(\mathsf{A}_\tau \wedge \neg\mathsf{A}_\tau) \rightarrow \mathsf{B}_\tau$ | (2) |
| (3) | $?_2\mathsf{A}_\tau \wedge \neg\mathsf{A}_\tau$ | | |
| (5) | $!_4\mathsf{A}_\tau$ | $?_{3/\text{left}}$ | (4) |
| (7) | $!_6\neg\mathsf{A}_\tau$ | $?_{3/\text{right}}$ | (6) |

*The Opponent wins*

The Proponent loses here because (according to the general negative literal rule) he is not allowed to attack move (7) using the Opponent's move (5).

Similarly, the literal rule blocks validity of $\bigwedge_x (\mathsf{A}_x \to (\neg \mathsf{A}_x \to \mathsf{B}_x))$ and the quantified forms of other non-paraconsistent formulae. Here again it is possible to define quantified literal dialogues for intuitionistic and classical logic.

Let us consider an example of a thesis which is not intuitionistically but classically winnable: A quantified literal dialogue in L-D$^{\mathrm{i}}$ for $\bigwedge_x \neg\neg \mathsf{A}_x \to \neg\neg \bigwedge_x \mathsf{A}_x$ runs as follows:

| | Opponent | Proponent | |
|---|---|---|---|
| | | $\bigwedge_x \neg\neg \mathsf{A}_x \to \neg\neg \bigwedge_x \mathsf{A}_x$ | (0) |
| (1) | $?_0 \bigwedge_x \neg\neg \mathsf{A}_x$ | $!_1 \neg\neg \bigwedge_x \mathsf{A}_x$ | (2) |
| (3) | $?_2 \neg \bigwedge_x \mathsf{A}_x$ | $\otimes$ | |
| | $\otimes$ | $?_3 \bigwedge_x \mathsf{A}_x$ | (4) |
| (5) | $?_{4/\tau}$ | | |
| (7) | $!_6 \neg\neg \mathsf{A}_n$ | $?_{1/\tau}$ | (6) |
| | $\otimes$ | $?_7 \neg \mathsf{A}_\tau$ | (8) |
| (9) | $?_8 \mathsf{A}_\tau$ | $\otimes$ | |

*The Opponent wins*

The Proponent loses in L-D$^{\mathrm{i}}$ because he is not allowed to defend himself against the attack of the Opponent in move (5)—the last Opponent's attack not already defended by the Proponent was stated in move (9).

The Proponent wins in L-D$^{\mathrm{c}}$ because the restriction mentioned above does not hold. Thus the Proponent can answer the attack of move (5) with move (10) in the following dialogue in (quantified) L-D$^{\mathrm{c}}$ and win:

| | Opponent | Proponent | |
|---|---|---|---|
| | | $\bigwedge_x \neg\neg \mathsf{A}_x \to \neg\neg \bigwedge_x \mathsf{A}_x$ | (0) |
| (1) | $?_0 \bigwedge_x \neg\neg \mathsf{A}_x$ | $!_1 \neg\neg \bigwedge_x \mathsf{A}_x$ | (2) |
| (3) | $?_2 \neg \bigwedge_x \mathsf{A}_x$ | $\otimes$ | |
| | $\otimes$ | $?_3 \bigwedge_x \mathsf{A}_x$ | (4) |
| (5) | $?_{4/\tau}$ | $!_5 \mathsf{A}_\tau$ | (10) |
| (7) | $!_6 \neg\neg \mathsf{A}_\tau$ | $?_{1/\tau}$ | (6) |
| | $\otimes$ | $?_7 \neg \mathsf{A}_\tau$ | (8) |
| (9) | $?_8 \mathsf{A}_\tau$ | $\otimes$ | |

*The Proponent wins*

It is possible to define tableaux for the winning strategies which correspond to these dialogue systems (see Rahman and Carnielli 1998). To obtain paraconsistent tableaux systems considered as an extension of those for DFL add the following restriction to the closing rules:[7]

Definition 6. Paraconsistent Restriction. *Check after finishing the tableau and before closing branches that for every elementary* P*-statement which follows from the application of an* O*-rule to the corresponding negative* O*-literal (i.e. for every attack on a negative* O*-literal) there is an application of a* P*-rule to a negative* P*-literal which yields an* O*-positive literal with the same atomic formula as the above-mentioned attack of the Proponent. Those elementary* P*-attacks on the corresponding negative* O*-literals which do not meet this condition cannot be used for closing branches and can thus be deleted.*

This approach to paraconsistency blocks triviality for the literal case only, that is, a thesis of the form $((a \wedge b) \wedge \neg(a \wedge b)) \rightarrow c$ is still valid. One way to see the literal rule is to think of it as distinguishing between the internal or copulative negation from the external or sentential negation.[8] That is, in the standard approaches to logic, the elementary proposition $\mathsf{A}_n$ has the internal logical form: $n \,\epsilon\, \mathsf{A}$ (where $\epsilon$ stands for the copula: $n$ *is* $\mathsf{A}$) and the negation of it the form: $n \,\epsilon'\, \mathsf{A}$ ($n$ *is not* $\mathsf{A}$). Now in this standard interpretation the negative copula is equivalent to the expression $\neg A$, where $A$ can also be complex. This equivalence ignores the distinction between the internal (copulative) form and the external or sentential form of elementary propositions. The literal approach to paraconsistency takes this distinction seriously, with the result that contradictions which cannot be carried on at the literal level should be freed of paraconsistent restrictions.

## 4. Conclusions

This article is one of a series based on the seminar "Erweiterungen der Dialogischen Logik" ("extensions to dialogical logic") held in Saarbrücken in the summer of 1998 by Shahid Rahman and Helge

[7]Although you can produce intuitionistic and classical free-paraconsistent systems the intuitionistic version seems more appropriate. Such a system is defended in Rahman 1999, where, following Read, (see Read 1994, p. 137) I support the idea that although classical logic might have some plausibility for existents it loses this plausibility for non-existents. In the paper mentioned I connect this argument with the theory of *privatio* of the Spanish philosopher Francisco Suárez (1548–1617) (see Suárez 1966, pp. 434–440).

[8]The difference between internal and external negation has been worked out for other purposes by A. A. Sinowjew (1970) and Wessels/Sinowjew (1975).

Rückert. The same seminar has motivated the publication of *The Dialogical Approach to Paraconsistency* by Rahman and Carnielli (1998), *On Dialogues and Ontology. The Dialogical Approach to Free Logic* by Rahman, Fischmann, and Rückert (1999), *Dialogische Modallogik für T, B, S4 und S5* (Rahman and Rückert 1999b) and *Dialogische Logik und Relevanz* (Rahman and Rückert 1998) and *Die Logik der zusammenhängenden Aussagen: ein dialogischer Ansatz zur konnexen Logik* (1999a) by Rahman and Rückert. One important aim of these articles (and the present paper) is to show how to build a common semantic language for different non-standard logics in such a way that 1. the semantic intuitions behind these logics can be made transparent, 2. combinations of these logics can be easily achieved, 3. a common basis is proposed for discussion of the philosophical consequences of these logics—the philosophical point here is to undertake the task of discussing the semantics of non-classical logics from a pragmatical point of view, one which commits itself neither to a correspondence theory of truth nor to a possible-world semantics.

One of the consequences of the dialogical approach is that two of the above-mentioned logics can be seen as extending the formal rule for elementary propositions, namely free and paraconsistent logics. This offers a perspective on these logics which seems to be close to Hugh MacColl's reflections on symbolic existence and demands a new concept of logical form. This new concept of logical form should allow valid and invalid forms to be differentiated without going back to a mere syntactic notion—but this is another interesting story.

## Acknowledgements

The results of this paper are the first achievements of a collaboration project between the Archives-Centre d'Etudes et de Recherches Henri Poincaré (Prof. Gerhard Heinzmann), Université Nancy 2 and FR 5.1 Philosophie (Prof. Kuno Lorenz), Universität des Saarlandes, supported by the Fritz-Thyssen-Stiftung which I wish to thank expressly.

I also wish to thank Manuel Bremer (Köln), Erik C. W. Krabbe (Groningen), Narahari Rao (Saarbrücken) and Helge Rückert (Saarbrücken) for their critical comments on earlier versions of this paper and Mrs. Cheryl Lobb de Rahman for her careful grammatical revision.

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