Recent developments in formal pragmatics

Part 3/3: Game Theory

The Arctic University of Norway, November 2018

Kjell Johan Sæbø

Note two facts about · Bidirectional Optimality Theory on the one hand and · the Grammatical theory on the other:

➔ In BiOT, something is going on behind the scenes; what one sees is a stable state in the addressee's perspective, and it does not really

> show how implicatures can be amplified or dampened when the agents reason about each other (Potts 2013).

➔ In GT, only Quantity implicatures, scalar implicatures in a wide sense, can be derived – it offers no way to derive, say, markedness implicatures. Besides, it has no systematic way to model the sensitivity of implicatures to factors like **message cost** and a **state's prior probability**.

A theory of **signaling games**, whether in terms of **iterated best response** or **lexical uncertainty**, is more explicit and more comprehensive.

1 Interlocutors as functions from each other to functions from $<\text{message}, \text{state}>$ pairs to reals

Let us go through the paradigm scalar implicature case from the first, BiOT installment, to see how a Game theoretic model makes a difference, where

> production and interpretation are modeled as a recursive process where the listener and speaker reason about each other reasoning about each other. (Potts 2013)

Let us begin with very simple definitions of the initial listener, the speaker and the listener, adapted from Potts (2013):[1]

(1) **Initial listener**

$$L_0(<m\,,\sigma>) = \frac{\mathcal{I}(\sigma \in [\![m]\!]) / |[\![m]\!]|}{\sum_{\sigma'} \mathcal{I}(\sigma' \in [\![m]\!]) / |[\![m]\!]|}$$

(2) **Speaker**

$$S(L)(<m\,,\sigma>) = \frac{L(<m\,,\sigma>)}{\sum_{m'} L(<m'\,,\sigma>)}$$

(3) **Listener**

$$L(S)(<m\,,\sigma>) = \frac{S(<m\,,\sigma>)}{\sum_{\sigma'} S(<m\,,\sigma'>)}$$

Now consider the recursive application of L to S to L to S to L_0:

$L_0(<y\,,x>)$	$\exists, < n$	$> n, \neg\forall$	$\forall$
sometimes	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
often	0	$\frac{1}{2}$	$\frac{1}{2}$
always	0	0	1

Table 1: 3 messages, 3 states, initial listener at a loss

$S(L_0)(<y\,,x>)$	$\exists, < n$	$> n, \neg\forall$	$\forall$
sometimes	1	$\frac{2}{5}$	$\frac{2}{11}$
often	0	$\frac{3}{5}$	$\frac{3}{11}$
always	0	0	$\frac{6}{11}$

Table 2: 3 messages, 3 states, $S(L_0)$

[1]Here we assume a flat prior over states and cost-free messages; $\mathcal{I}$ is a valuation function returning 1 if the argument is true, 0 otherwise; σ' ranges over the alternative states.

$L(S(L_0))(<y\,,x>)$	$\exists\,,<n$	$>n,\neg\forall$	$\forall$
sometimes	$\frac{55}{87}$	$\frac{22}{87}$	$\frac{10}{87}$
often	0	$\frac{11}{16}$	$\frac{5}{16}$
always	0	0	1

Table 3: 3 messages, 3 states, $L(S(L_0))$

$S(L(S(L_0)))(<y\,,x>)$	$\exists\,,<n$	$>n,\neg\forall$	$\forall$
sometimes	1	$\frac{352}{1309}$	$\frac{160}{1987}$
often	0	$\frac{957}{1309}$	$\frac{435}{1987}$
always	0	0	$\frac{800}{1987}$

Table 4: 3 messages, 3 states, $S(L(S(L_0)))$

$L(S(L(S(L_0))))(<y\,,x>)$	$\exists\,,<n$	$>n,\neg\forall$	$\forall$
sometimes	$\frac{2600983}{3490347}$	?	?
often	0	?	?
always	0	0	1

Table 5: 3 messages, 3 states, $L(S(L(S(L_0))))$

The listener's confidence in the implicature grows logarithmically but does not reach certainty. It can, however, be affected, positively or negatively, by

- differing costs attached to the messages,
- differing prior probabilities attached to the states.

2 Varying costs and priors: two cases

Let us look at two difficult cases where adding costs and priors helps.

2.1 Q-implicatures and the symmetry problem

According to Bergen, Levy and Goodman (2016), the symmetry problem is "a problem with constructing the scales for the implicature computations:

> there are multiple consistent ways of constructing the scales, and different scales will give rise to different implicatures."

Because the only formal requirement on a scale is that items higher on it be logically stronger than those lower on it, a possible scale for *some* is

> <"some", "some but not all">. If this scale is used, "some" will imply that "some but not all" is not true, i.e., that "all" is true.

These authors break this symmetry by assigning *some but not all* a much greater cost than *some*. Formally, we need to introduce the 'initial speaker' and the cost function C, ranging over $[\,0\,,1\,]$, in the definition of the speaker:

(4) **Initial speaker**

$$S_0(<m\,,\sigma>) = \frac{(\mathcal{I}(\sigma \in [\![m]\!])/\,|\,m : \sigma \in [\![m]\!]\,|)\,C(m)}{\sum_{m'}(\mathcal{I}(\sigma \in [\![m']\!])/\,|\,m : \sigma \in [\![m]\!]\,|)\,C(m')}$$

(5) **Speaker**

$$S(L)(<m\,,\sigma>) = \frac{L(<m\,,\sigma>)\,C(m)}{\sum_{m'} L(<m'\,,\sigma>)\,C(m')}$$

$S_0(<y\,,x>)$	$\exists\,,\neg\forall$	$\forall$
some	$\frac{2}{3}$ $(\frac{1}{2})$	$\frac{1}{2}$
some, not all	$\frac{1}{3}$ $(\frac{1}{2})$	0
all	0	$\frac{1}{2}$

Table 6: S_0, $C(\mathit{some}) = C(\mathit{all}) = 1$, $C(\mathit{somenotall}) = .5$ (0)

Consider now the recursive application of L to S to L to S_0:

$L(S_0)(<y\,,x>)$	$\exists\,,\neg\forall$	$\forall$
some	$\frac{4}{7}$ $(\frac{1}{2})$	$\frac{3}{7}$ $(\frac{1}{2})$
some, not all	1	0
all	0	1

Table 7: $L(S_0)$, $C(some) = C(all) = 1$, $C(somenotall) = .5$ (0)

$S(L(S_0))(<y\,,x>)$	$\exists\,,\neg\forall$	$\forall$
some	$\frac{8}{11}$ $(\frac{1}{3})$	$\frac{3}{10}$ $(\frac{1}{3})$
some, not all	$\frac{3}{11}$ $(\frac{2}{3})$	0
all	0	$\frac{7}{10}$ $(\frac{2}{3})$

Table 8: $S(L(S_0))$, $C(some) = C(all) = 1$, $C(somenotall) = .5$ (0)

$L(S(L(S_0)))(<y\,,x>)$	$\exists\,,\neg\forall$	$\forall$
some	$\frac{80}{113}$ $(\frac{1}{2})$	$\frac{33}{113}$ $(\frac{1}{2})$
some, not all	1	0
all	0	1

Table 9: $L(S(L(S_0)))$, $C(some) = C(all) = 1$, $C(somenotall) = .5$ (0)

We see that as long as the signalling cost of the message *some, not all* is not taken into account, the listener is just as likely to interpret the message *some* as 'all' as as 'some but not all', but as soon as a .5 factor cost is calculated, *some* is more and more likely to be interpreted as 'some but not all' again.

2.2 I-implicatures and the tension with Quantity

In the framework of the **Rational Speech Act** model (Frank and Goodman 2012, Bergen, Levy and Goodman 2016), Poppels and Levy (2015) address how to balance the pressure to strengthen an expression to exclude

- a stronger alternative (Q-based implicatures)

and the pressure to strengthen an expression to exclude

- atypical cases (R-based or Informativeness implicatures).

Case in point: the indefinite article can be strengthened in two ways:

- to something possessive or
- to something anti-possessive.

(6) lends itself to a Q-implicature while (7) lends itself to an R-implicature:

(6) He was in a bed. $\rightsquigarrow$ not his own bed

(7) I broke a toe yesterday. $\rightsquigarrow$ one of my own toes

In both cases, *a* competes with the same-cost, stronger alternative *his*/*my* – but in (7), this is counterbalanced by the OWN'S higher **prior probability**.

Let us introduce the 'initial listener' L_0 and the prior probability function P, ranging over $[\,0\,,1\,]$, in the definition of L:

(8) **Initial listener**

$$L_0(<m\,,\sigma>) = \frac{(\mathcal{I}(\sigma \in [\![m]\!])/\,|\,m : \sigma \in [\![m]\!]\,|)\;P(\sigma)}{\sum_{\sigma'}(\mathcal{I}(\sigma' \in [\![m]\!])/\,|\,m : \sigma \in [\![m]\!]\,|)\;P(\sigma')}$$

(9) **Listener**

$$L(S)(<m\,,\sigma>) = \frac{S(<m\,,\sigma>)\;P(\sigma)}{\sum_{\sigma'} S(<m\,,\sigma'>)\;P(\sigma')}$$

Assume that P(OTHER'S)$=.2$ and P(OWN)$=.8$ if you break a toe, and consider the recursive application of L to S to L_0 on the next page.

Note that $<$ *a toe* , OTHER'S $>$ approaches .5 listener confidence, more and more slowly. It never reaches implicature level.

$L_0(<y,x>)$	OTHER'S	OWN
a toe	$\frac{1}{5}$	$\frac{4}{5}$
my toe	0	1

Table 10: L_0, P(OTHER'S)$=.2$ and P(OWN)$=.8$

$S(L_0)(<y,x>)$	OTHER'S	OWN
a toe	1	$\frac{4}{9}$
my toe	0	$\frac{5}{9}$

$L(S(L_0))(<y,x>)$	OTHER'S	OWN
a toe	$\frac{9}{25}$	$\frac{16}{25}$
my toe	0	1

$S(L(S(L_0)))(<y,x>)$	OTHER'S	OWN
a toe	1	$\frac{16}{41}$
my toe	0	$\frac{25}{41}$

$L(S(L(S(L_0))))(<y,x>)$	OTHER'S	OWN
a toe	$\frac{41}{105}$	$\frac{64}{105}$
my toe	0	1

Table 11: $L(S(L(S(L_0)))$, P(OTHER'S)$=.2$ and P(OWN)$=.8$

3 Lexical uncertainty

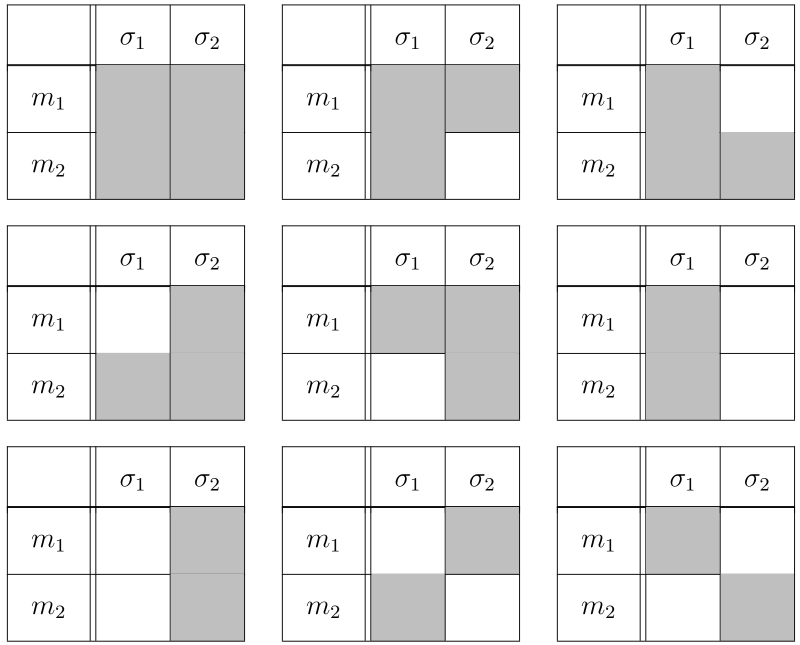
Scientists like Potts et al. (2016) and Bergen et al. (2016) assume that word meanings are not fixed across speakers and contexts: discourse participants do not share one lexicon but consider many lexica and synthesize them.

> The guiding idea is that, in interaction, pragmatic agents reason about possible refinements of their lexical items, with the base lexical meaning serving as a kind of anchor to which each word's interpretation is loosely tethered. (Potts et al. 2016)

The lexical uncertainty version of the rational speech model is essential for deriving **M-implicatures** (marked expression ⇔ marked interpretation).

Slightly simplified, this is how:

1. The listener is uncertain what the cost-free form m_1 and the costly form m_2 mean, the likely σ_1 or the unlikely σ_2. She considers nine lexica:



Then she calculates the utility values for each pairing over all these lexica, taking the prior probabilities into account, say, .8 for σ_1 and .2 for σ_2:

	σ_1	σ_2
m_1	$\frac{4}{5}$	$\frac{1}{5}$
m_2	$\frac{4}{5}$	$\frac{1}{5}$

	σ_1	σ_2
m_1	$\frac{4}{5}$	$\frac{1}{5}$
m_2	1	0

	σ_1	σ_2
m_1	1	0
m_2	$\frac{4}{5}$	$\frac{1}{5}$

	σ_1	σ_2
m_1	$\frac{4}{5}$	$\frac{1}{5}$
m_2	0	1

	σ_1	σ_2
m_1	0	1
m_2	$\frac{4}{5}$	$\frac{1}{5}$

	σ_1	σ_2
m_1	1	0
m_2	1	0

	σ_1	σ_2
m_1	0	1
m_2	0	1

	σ_1	σ_2
m_1	1	0
m_2	0	1

	σ_1	σ_2
m_1	0	1
m_2	1	0

Subsequently, the speaker calculates the utility values over all these values, taking the costs into account, say, the factor .5 for m_2, and in a next step, the listener averages over these values to compute her utility values:

L_1	σ_1	σ_2
m_1	.81	.19
m_2	.79	.21

There is a certain association between costly form and unlikely content here, and this association can be strengthened by

- more response iterations,
- increasing the degree of "greedy rationality", encoded in parameter λ,
- including the "null utterance" true in all states and very costly.

It is an open question, though, whether this is more explanatory than the notion of weak optimality introduced by Blutner (2000).

References

Benz, A., G. Jäger and R. v. Rooij (eds.) (2005). *Game Theory and Pragmatics*. Basingstoke, Hampshire: Palgrave McMillan.

Bergen, L., R. Levy, N. Goodman (2016). Pragmatic reasoning through semantic inference. *Semantics and Pragmatics*, 9(20). http://dx.doi.org/10.3765/sp.9.20.

Chierchia, G., D. Fox and B. Spector (2012). The grammatical view of scalar implicatures and the relationship between semantics and pragmatics. In Maienborn, von Heusinger and Portner (eds.), *Semantics: An international handbook of natural language meaning*, vol 3 (pp. 2297–2332), Berlin: Mouton de Gruyter.

Fox, D. and R. Katzir (2011). On the characterization of alternatives. *Natural Language Semantics*, 19(1), 87–107. doi: 10.1007/s11050-010-9065-3.

Frank, M. and N. Goodman (2012). Predicting pragmatic reasoning in language games. *Science*, 336(6084), 998.

Franke, M. (2009). *Signal to act: Game theory in pragmatics*. PhD dissertation, Universiteit van Amsterdam.

Franke, M. and G. Jäger (2014). Pragmatic back-and-forth reasoning. In S. P. Reda (ed.), *Pragmatics, semantics and the case of scalar implicatures* (pp. 170–200), London: Palgrave Macmillan.

Horn, L. R. (1984). Towards a new taxonomy for pragmatic inference: Q-based and R-based implicatures. In D. Schiffrin (ed.), *Meaning, Form and Use in Context: Linguistic Applications* (pp. 11–42), Washington: Georgetown University Press.

Jäger, G. (2012). Game theory in semantics and pragmatics. In Maienborn, von Heusinger and Portner (eds.), *Semantics: An international handbook of natural language meaning*, vol 3 (pp. 2487–2425), Berlin: Mouton de Gruyter.

Poppels, T. and R. Levy (2015). Resolving Quantity and Informativeness implicature in indefinite reference. In T. Brochhagen, F. Roelofsen and N. Theiler (eds.), *Proceedings of the 20th Amsterdam Colloquium* (pp. 313–322), Amsterdam: ILLC.

Potts, C. (2013). Conversational implicature: interacting with grammar. Ms., Stanford University.

Potts, C., D. Lassiter, R. Levy and M. C. Frank (2016). Embedded implicatures as pragmatic inferences under compositional lexical uncertainty. *Journal of Semantics*, 33(4), 755–802.

van Rooij, R. and M. Franke (2015). Optimality-theoretic and Game-theoretic approaches to implicature. *Stanford Encyclopedia of Philosophy*.